



Oxford Cambridge and RSA

Monday 3 June 2019 – Morning

A Level Further Mathematics A

Y540/01 Pure Core 1

Time allowed: 1 hour 30 minutes



You must have:

- Printed Answer Booklet
- Formulae A Level Further Mathematics A

You may use:

- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total mark for this paper is **75**.
- The marks for each question are shown in brackets [].
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **8** pages.

Answer **all** the questions.

1 In this question you must show detailed reasoning.

The quadratic equation $x^2 - 2x + 5 = 0$ has roots α and β .

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. [1]

(b) Hence find a quadratic equation with roots $\alpha + \frac{1}{\beta}$ and $\beta + \frac{1}{\alpha}$. [3]

2 Indicate by shading on an Argand diagram the region

$\{z : |z| \leq |z - 4|\} \cap \{z : |z - 3 - 2i| \leq 2\}$. [3]

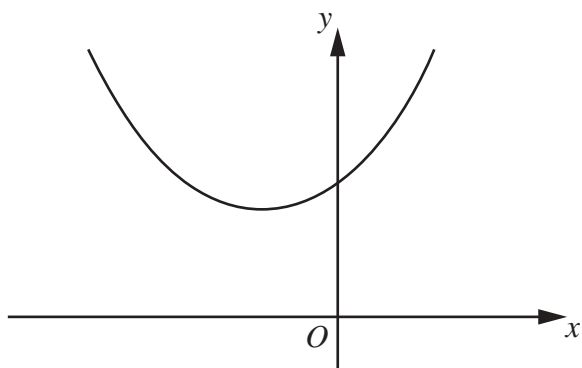
3 In this question you must show detailed reasoning.

You are given that $x = 2 + 5i$ is a root of the equation $x^3 - 2x^2 + 21x + 58 = 0$.

Solve the equation. [4]

4 Using the formulae for $\sum_{r=1}^n r$ and $\sum_{r=1}^n r^2$, show that $\sum_{r=1}^{10} r(3r - 2) = 1045$. [3]

- 5 The diagram shows part of the curve $y = 5 \cosh x + 3 \sinh x$.



- (a) Solve the equation $5 \cosh x + 3 \sinh x = 4$ giving your solution in exact form. [4]
- (b) **In this question you must show detailed reasoning.**

Find $\int_{-1}^1 (5 \cosh x + 3 \sinh x) dx$ giving your answer in the form $ae + \frac{b}{e}$ where a and b are integers to be determined. [3]

- 6 You are given that $y = \tan^{-1} \sqrt{2x}$.

(a) Find $\frac{dy}{dx}$. [2]

(b) Show that $\int_{\frac{1}{6}}^{\frac{1}{2}} \frac{\sqrt{x}}{(x+2x^2)} dx = k\pi$ where k is a number to be determined in exact form. [4]

- 7 The function $\operatorname{sech} x$ is defined by $\operatorname{sech} x = \frac{1}{\cosh x}$.

(a) Show that $\operatorname{sech} x = \frac{2e^x}{e^{2x} + 1}$. [2]

(b) Using a suitable substitution, find $\int \operatorname{sech} x dx$. [4]

- 8 The equation of a plane is $4x + 2y + z = 7$.
The point A has coordinates $(9, 6, 1)$ and the point B is the reflection of A in the plane.

Find the coordinates of the point B .

[6]

9 **In this question you must show detailed reasoning.**

You are given the complex number $\omega = \cos \frac{2}{5}\pi + i \sin \frac{2}{5}\pi$ and the equation $z^5 = 1$.

(a) Show that ω is a root of the equation. [2]

(b) Write down the other four roots of the equation. [1]

(c) Show that $\omega + \omega^2 + \omega^3 + \omega^4 = -1$. [2]

(d) Hence show that $\left(\omega + \frac{1}{\omega}\right)^2 + \left(\omega + \frac{1}{\omega}\right) - 1 = 0$. [3]

(e) Hence determine the value of $\cos \frac{2}{5}\pi$ in the form $a + b\sqrt{c}$ where a , b and c are rational numbers to be found. [4]

10 You are given the matrix \mathbf{A} where $\mathbf{A} = \begin{pmatrix} a & 2 & 0 \\ 0 & a & 2 \\ 4 & 5 & 1 \end{pmatrix}$.

(a) Find, in terms of a , the determinant of \mathbf{A} , simplifying your answer. [2]

(b) Hence find the values of a for which \mathbf{A} is singular. [2]

You are given the following equations which are to be solved simultaneously.

$$ax + 2y = 6$$

$$ay + 2z = 8$$

$$4x + 5y + z = 16$$

(c) For each of the values of a found in part (b) determine whether the equations have

- a unique solution, which should be found, or
- an infinite set of solutions or
- no solution.

[7]

- 11** A particle is suspended in a resistive medium from one end of a light spring. The other end of the spring is attached to a point which is made to oscillate in a vertical line.

The displacement of the particle may be modelled by the differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = 10 \sin t$$

where x is the displacement of the particle below the equilibrium position at time t .

When $t = 0$ the particle is stationary and its displacement is 2.

- (a) Find the particular solution of the differential equation. **[11]**
- (b) Write down an approximate equation for the displacement when t is large. **[2]**

END OF QUESTION PAPER

BLANK PAGE



Oxford Cambridge and RSA

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact The OCR Copyright Team, The Triangle Building, Shaftesbury Road, Cambridge CB2 8EA.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.